Anders Ångström and His Early Papers on Probability Forecasting and the Use/Value of Weather Forecasts

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Abstract

Anders K. Ångström was known primarily for his contributions to the field of atmospheric radiation. However, his scientific interests encompassed many diverse topics. This paper describes the contents of two early, remarkable, and, until recently, largely unknown papers by Ångström on probability forecasting and the use/value of weather forecasts. These papers, entitled "Sannolikhet och Praktisk Väderleksprognos" ("Probability and Practical Weather Forecasting") and "On the Effectivity of Weather Warnings," were published in 1919 and 1922, respectively.

Noteworthy features of these two papers include 1) a discussion of the sources of uncertainty in weather forecasting, 2) discourses on the problems of estimating probabilities by means of empirical relative frequencies and forecasters' subjective judgments, 3) the use of a Gaussian model to describe the accuracy of minimum temperature forecasts, 4) the identification of the ratio of the protection costs to the "risk value" if protective action is not taken as a characteristic of users of forecasts, 5) analytical expressions for the economic value of weather warnings, 6) quantitative analysis of the problems faced by forecasters in deciding whether or not to issue weather warnings when they are uncertain about future weather conditions, and 7) arguments concerning the need to obtain estimates of the costs and losses that may be incurred by potential users in order to assess economic effectiveness.

The contents of Ångström's two papers are reviewed and summarized, making extensive use of quotations from the texts. An effort is made to place the papers and their contents in proper historical context. Two topics of current interest, namely, ensemble forecasting and the provision of specialized weather services, are discussed briefly in light of the results presented and issues raised in these papers.

1. Introduction

For some purposes, it may be convenient to view probability forecasting (i.e., the quantification of un-

certainty in forecasts) and the use/value of forecasts (e.g., optimal use and economic value of forecasts) as distinct topics and to study them separately. In reality, however, these topics are linked in fundamental ways. For example, forecasts expressed in a probabilistic format generally are more useful and valuable to users than forecasts expressed in a categorical (i.e., nonprobabilistic) format (e.g., see Thompson 1962; Murphy 1977). Moreover, prescriptive studies of the use/value of forecasts reveal that users need probabilistic information (specifically, the conditional probabilities of the events of interest given the forecasts) to make the best possible use of the forecasts (e.g., see Winkler and Murphy 1985; Katz and Murphy 1994).

The first published papers related to probability forecasting and the value of forecasts appeared in meteorological literature during the first 25 years of this century. In the case of probability forecasting, the names of Cooke (1906a,b), von Myrbach (1913), and Hallenbeck (1920) are usually associated with this early work. In the case of the use/value of forecasts, Bilham (1922) generally is credited with publishing the first paper specifically devoted to this topic. In these early papers, the topics of probability forecasting and forecast use/value were not linked in any significant way.

Although several individuals have made serious efforts to trace the historical development of these topics (e.g., Murphy and Allen 1970), it is only recently that two remarkable papers on these topics by the Swedish meteorologist Anders Ångström have come to light. The first paper appeared in 1919 in a weekly technological magazine, and the second paper was published in 1922 in a statistical journal. Although the first paper focused on probability forecasting and the second paper concentrated on the use/value of forecasts, the interrelationship between these topics is readily apparent and emphasized in both texts. These papers were written by Ångström early in his career; he went on to become well known for his contributions to the field of atmospheric radiation.

The primary purposes of this paper are to describe

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the contents of Ångström's two papers and to place them and their contents in proper historical context. In addition, it seems appropriate to relate the issues addressed in Ångström's papers to some topics of current interest in the forecasting area. Two such topics are considered here, namely, ensemble forecasting and the provision of specialized weather services.

A brief sketch of Anders Ångström's career is included in section 2. This section also introduces the two papers of particular interest. Sections 3 and 4 summarize the principal issues addressed, basic results obtained, and key points made in the 1919 and 1922 papers, respectively. An attempt is made in section 5 to put Ångström's papers in proper historical context and to relate these papers to issues of current interest. Section 6 consists of a brief summary and some concluding remarks.

2. Anders Ångström

Anders Ångström was born in February 1888 and died in his 94th year in October 1981. His father and grandfather were professors of physics at the University of Uppsala. The grandfather, Anders J. Ångström (1814–1874), made the name Ångström known throughout the world when the basic unit of the wavelength of light was named after him. The father, Knut J. Ångström (1857–1910), was also a well-known radiation physicist. He designed the compensation pyrheliometer, which was used throughout the world for many years to measure direct solar radiation.

Anders K. Ångström obviously came of good stock, since physics and radiation ran in the family! It was natural that Anders should also study radiation and continue the line of his ancestors. His doctoral thesis was entitled "A Study of the Radiation of the Atmosphere," and it was presented at Uppsala in 1916. Some years before he had participated in an expedition to Algeria and had led a study at Mount Whitney in California, both involving the measurement of atmospheric radiation. His interest in this topic brought him closer to the meteorological community, and on 1 January 1919 Anders Ångström was employed at the Statens Meteorologiska och Hydrografiska Anstalt (SMHA), which at that time was the name of the Swedish Meteorological and Hydrological Institute (SMHI). From that day meteorologists and hydrologists worked side by side in the same institute.

During the first year in his new position, Ångström took an interest in practical weather forecasting. The second paper that he wrote at SMHI was entitled "Sannolikhet och Praktisk Väderleksprognos" ("Probability and Practical Weather Forecasting") (Ångström

1919). This paper, which was published as a contribution to a weekly magazine for technologists, appears to represent the text of a lecture. Shortly thereafter, Ångström's continuing and lifelong interest in atmospheric radiation led him to conduct some research related to the problem of frost and its connection with nighttime radiation. Through this work, in turn, he was introduced to the practical problems of agriculture and the influence of weather and weather forecasts on farmers' activities and decisions. This interest culminated in 1922 in the publication of a paper entitled "On the Effectivity of Weather Warnings," which appeared in the first volume of *Nordisk Statistisk Tidskrift* (Ångström 1922). It is these two papers that are the primary focus of this article.

Ångström's interests ranged over many areas, even if his most widely recognized contributions occurred in the field of atmospheric radiation. In an interview conducted by H. Taba with notable assistance provided by A. Nyberg, and published in the *WMO Bulletin* in 1982 (World Meteorological Organization 1982), the 92-year-old Ångström mentioned "the efficiency of weather forecasts" as one of several topics on which he worked in those early days. During the decade of the 1920s he also wrote several papers on the relationships among radiation, cloudiness, and frost. It is also interesting to note that evidently (see Glantz 1991) the first known use of the term "teleconnection" was by Ångström in a paper (Ångström 1935) in which he discussed the atmospheric "seesaw" mechanism now referred to as the North Atlantic oscillation.

Even earlier, through his good friend C.-G. Rossby, Ångström was able to obtain insight into the new theories and practices in weather analysis and forecasting that were being developed at the time in Bergen, Norway. With the topics of probability forecasting and forecast use/value in mind, it should be noted that Rossby evidently read Ångström's manuscript "On the Effectivity of Weather Warnings" and contributed some useful comments (see section 4).

During the last years of his long career at SMHI (namely, 1949–54), Ångström served as "Överdirektör" of the institute. During this time, he offered Rossby, who in the meantime had become a leading meteorologist in the United States, a special post as scientific advisor to SMHI. In their private correspondence [reviewed by Erik Liljas (EL)] Rossby gives advice concerning the establishment of modern aerological stations and the future prediction of the weather based on mathematical calculations with computing machines. Rossby then returned to Stockholm and continued to play a leading role in modern meteorology.

The career of Ångström did not end with his retirement from SMHI at the end of 1954. For example, he

made frequent visits to the Eppley Laboratory in Newport, Rhode Island, where he worked on the development and refinement of instruments and the introduction of empirical methods of estimating relationships among meteorological parameters.

During Ångström's long life, he was involved in many international activities and missions. As a result of his significant contributions to and international leadership in the area of atmospheric radiation, he was awarded the International Meteorological Organization (IMO) prize in 1962. In Sweden, his contributions to climatology and his popular articles and seminars on storm events and practical meteorological problems also made the name Ångström well known in many circles outside of meteorology. A portrait of Anders Ångström, sketched by one of the authors (EL) from a photograph taken in 1978 around the time of his 90th birthday, appears in Fig. 1.

3. Sannolikhet och Praktisk Väderleksprognos (Probability and Practical Weather Forecasting) (1919)

This paper begins with a comparison of the prediction of astronomical and meteorological events. Ångström notes that astronomical events can be predicted with a very high probability, but that "when we are dealing with weather events, these will always be afflicted with a certain degree of uncertainty." In support of this point, he indicates that current short-range forecasts of rainfall and high winds are correct only about 75% and 55% of the time, respectively. This statement demonstrates that Ångström recognized the existence of a relationship between the a priori uncertainty inherent in weather forecasts and their a posteriori degree of correctness.

Ångström attributes this uncertainty to three factors. First, a very large number of elements must be observed to determine the state of the atmosphere. Second, the basic instability of the atmosphere must be considered. Ångström uses the example (from mechanics) of a cone balancing on its top and points out that even the smallest inaccuracy in measuring the orientation of the axis of the cone from the vertical may lead to a completely erroneous prediction as to the direction in which it subsequently will fall. He summarizes this argument by indicating that "(t)he circumstance that a very feeble condition might give cause to great implications is one of the basic reasons behind the difficulties in the prediction of the state of the atmosphere." Third, Ångström mentions the problem of calculating future states of the atmosphere even from a well-defined initial state. In this regard, he refers to V. Bjerknes, who had indicated that in some situa-

FIG. 1. A portrait of Anders K. Ångström sketched by E. Liljas from a photograph taken in 1978.

tions it is theoretically possible to predict the weather, but that a forecast three days in advance would require a month's calculations. Ångström concludes this discussion by stating that "(t)oday it is probably apparent that both theoretical and statistical meteorology has importance for weather prediction, and that only cooperation and interaction between the two will lead to progress in the future."

Ångström then turns to the problem of estimating the probability of weather events based on an empirical approach. He adopts a traditional statistical perspective here, in the sense that the probability of an event is estimated by the relative frequency with which the event of interest occurs in a sample of situations defined by specified values of relevant weather variables ("prognostic factors"). He notes that the search for such prognostic factors is an important mission of theoretical and experimental meteorology.

Two examples are used by Ångström to illustrate the fact that the likelihood that a weather prediction will be correct (or, equivalently, that the probability that the event of interest will occur) varies from situation to situation, depending on the values of such prognostic factors. The first example is concerned with forecasts

of the occurrence of rain in Stockholm in the summer, using data presented by Rolf (1917). In this case, the likelihood of rain in a 24-h period beginning at 0800 is determined as a function of the barometric pressure at 2100 the previous evening, with the probability of rain increasing monotonically with decreasing pressure.

The second example, which is developed in some detail, relates to the prediction of frost. In this case, Ångström describes a statistical model based on the Gaussian distribution that estimates the probability of frost (actually the probability that the minimum temperature is below freezing) as a function of the previous evening's wet-bulb temperature. By integrating the Gaussian density function, Ångström is able to compute the probability of frost for various wet-bulb temperatures, and he presents some results of these calculations in both tabular and graphical form.

Ångström then indicates that whenever "practical actions are expected in connection with the prediction, other problems appear" (i.e., problems other than simply estimating the likelihood of various weather conditions). These problems involve "a factor that simply can be named the ratio between protection cost and risked value" and relate to the conditions under which forecasts of events such as storms, frost, etc., should be issued. He notes that "(a) meteorological prediction that is issued ignoring the conditions within the economy and commerce where it is supposed to be of importance may easily cause more loss than profit" and that "(t)he most effective forecast is rarely the one that is connected with high probabilities."

An example involving wet-bulb temperatures and the probability of frost (introduced earlier) is used by Ångström to illustrate the conditions under which a frost warning should be issued. For instance, he points out that if the protection cost a is very small compared to the value risked when no preventive actions are taken b , then a "frost warning should be issued even if the risk for frost is very modest." Ångström indicates that in general protective actions should be taken whenever the probability of the event of interest exceeds the ratio of a to b . [Note that approximately 25 years later this ratio was given the name "cost-loss ratio," but Ångström does not use this terminology here or in Ångström (1922).] Interestingly, he also remarks that the issuance of warnings (in this case, German storm warnings) when the probability of a storm is less than 0.50 may be entirely appropriate from the point of view of users of the forecasts, but at the same time may lead to a relatively large number of incorrect forecasts.

Finally, Ångström turns to the problem of the preparation of weather forecasts and offers two possible approaches to this problem. First, the forecaster could

issue a warning (or not) based on his own opinion regarding the magnitude of the ratio of a to b . Here, however, he points out the difficulties associated with the fact that this ratio varies from season to season and location to location. The alternative approach, "which is certainly to be recommended" according to Ångström, is "to issue warnings according to a certain scale in which a specific formulation of the warning corresponds to a specific probability range for the occurrence." He then briefly discusses practical issues related to the identification of such scales for different weather events (e.g., frost, summer rain, storm winds). Some confusion arises in this discussion between scales that characterize levels of intensity of the phenomena of interest and scales that describe levels of uncertainty concerning the occurrence of the phenomena.

The paper concludes with a brief discussion of SMHA's efforts to introduce a frost forecasting program based on these considerations. To design such a program properly, SMHA sought information from farmers and others concerning the occurrence and intensity of frosts in their area, as well as information regarding the types of crops or plants damaged, the economic loss incurred, and the total value of the crops/plants at risk. Clearly, this program represented an extremely ambitious and far-sighted endeavor for its time.

4. On the effectivity of weather warnings (1922)

This paper contains three sections. Section 1 is concerned with the economic value of storm warnings, section 2 describes the relationship between the a priori probability of the correctness of forecasts and the effectiveness of warnings, and section 3 discusses the problems encountered by forecasters who issue categorical warnings and proposes a solution to these problems.

In section 1 Ångström investigates the "economical effectivity" of forecasts in the context of a situation involving warnings for storms and gales on the Swedish coast. He refers to the forecasts of interest, namely, forecasts of storm occurrence or nonoccurrence (i.e., storm/no storm forecasts), as "categorical forecasts." Ångström then introduces notation to describe the frequencies with which the various combinations of forecast and observed events occur in a hypothetical data sample (i.e., the elements of a 2×2 contingency table). In his notation, N denotes the number of occasions with storms, N_w denotes the number of occasions on which a warning is issued and a storm occurs, and N_w' denotes the number of occasions on which a

warning is issued but no storm occurs. As before, Ångström denotes the cost of protection in the case of a warning by a and the average "value lost" if a storm is not preceded by a warning (i.e., protective action is not taken) by b .

Ångström combines these frequencies and costs/losses to determine an expression for the "total economical output" when warnings are issued and used. This expression, denoted by K , is as follows:

$$K = (N_w + N_w')a + (N - N_w)b. \quad (1)$$

As a standard against which to compare K in (1), Ångström considers the economical output when no warnings are issued. Denoting this expression by K_1 , he writes

$$K_1 = Nb, \quad (2)$$

since $N_w = N_w' = 0$ in this case. Ångström then defines $\varepsilon = K_1/K$ as the "effectivity" of the warning system, with larger values of ε indicating greater effectivity. He notes that ε depends on the ratio of a to b but not on their respective individual magnitudes, thereby indicating that "it is ... not influenced by such factors as the size of the fleet (of ships) or the number of persons dependent on the forecasts."

The economical gain provided by the warning system is defined by Ångström to be the difference between K_1 in (2) and K in (1). Denoting the economical gain per unit storm by E , Ångström writes

$$E = (N_w/N)b - [(N_w + N_w')/N]a. \quad (3)$$

Since E is positive only if the value of the first term in (3) exceeds the value of the second term, Ångström states that "the warning system ... (is) ... useful" if

$$N_w/(N_w + N_w') > a/b. \quad (4)$$

In words, the inequality in (4) indicates that the warning system is useful if the ratio of storms correctly warned to the total number of warnings issued is greater than the ratio of a to b .

To illustrate the practical application of these concepts, Ångström used some results taken from Wallén (1921), who had recently developed and tested a new measure to assess the degree of correctness of storm warnings in Sweden. This measure was defined in terms of four conditional probabilities derived from the (joint and marginal frequencies associated with the) basic 2×2 contingency table. Introduction of these probabilities into (3) led to the following expression:

$$E = P_1 b - (P_1/P_3) a, \quad (5)$$

where P_1 is the probability of a warning given a storm and P_3 is the probability of a storm given a warning. As noted by Ångström, the expression for E in (5) indicates that only two of Wallén's four probabilities (in addition to a and b) must be specified to determine the economic effectiveness of the forecasts.

Ångström then used values of the conditional probabilities P_1 and P_3 reported by Wallén for various stations in Sweden to investigate the economic effectiveness of its storm warning system. Fixing the value of a , he found that warning systems for which b is at least twice as large as a were economically effective. He then argued that values of the ratio of a to b less than one-half must be the case for forecasts involving such events.

The problem of obtaining at least an approximate idea of the values of a and b was addressed next by Ångström. He indicated that "the most rational (approach) seems to be an inquiry among shipowners and fishermen." Moreover, he emphasized the importance of acquiring information about the ratio of a to b since "it is impossible to form without it an idea of the effectivity of the system." Ångström then went on to make the fundamental point (to which he returns in section 3) that "the principles according to which categorical warnings themselves are sent out ought to be made dependent on the value ... given to a/b ."

To get some idea of the relative values of a and b , Ångström adopted the working hypothesis that it is equally important not to overwarn as it is not to underwarn (he admitted that opinions regarding this hypothesis were somewhat divergent). On the basis of this assumption, he was able to derive an expression in which the ratio a/b was expressed in terms of the conditional probabilities P_1 and P_3 . Wallén's storm warning verification data were then used to obtain estimates of a/b as well as estimates of the effectivity of the warnings. These estimates suggested that the loss b was approximately 3.6 times the cost a and that the economical losses would have been about 1.4 times larger if no warnings had been issued. Finally, he states that since the economic values involved amounted to millions of Swedish kroner, "the economical profits (provided by the forecasts) may be considerable."

In section 2, the problem of the conditions under which a storm warning should be issued were examined from a different but related and more general point of view. First, Ångström considered an idealized forecaster with unlimited experience concerning the meteorological conditions (defined in terms of the values of various meteorological elements) under which the storms of interest do and do not occur. On a particular occasion this forecaster might recall that in n similar situations, storms actually occurred at the

location of interest on m occasions. Ångström then compared the expense when warnings are issued on all n occasions, na , with the expense when no warnings at all are issued, mb (recall the definitions of the cost a and the loss b). Then defining the ratio m/n as the probability P of a storm, Ångström indicated that it is economical (in the long run) to issue a warning when $P > a/b$. It should be noted that this inequality is identical in form to the inequality involving the cost-loss ratio that first appeared in the published literature some 25 years later!

Ångström then addressed the possible criticism that the probability of a storm could not be determined in such a relatively simple and straightforward manner in real-world situations. His words are worth quoting at some length here: "It is easily seen that this objection is only valid as far as it denies the possibility of determining P according to the method indicated above. But it seems scarcely necessary to give to P this narrow definition previously indicated. We may define P simply as resulting from the combined experience, judgement, intuition, computation and knowledge of the forecaster. In fact the evaluation of P enters as a fundamental, but perhaps often too less conscious condition for forecasting in general." In these sentences, Ångström appears to be arguing for a subjective interpretation of probability, in which the probability is based on both the forecaster's knowledge, experience, etc., as well as any other information, such as "computations," that may be available.

To illustrate these ideas Ångström considered once again the problem of frost prediction. In this case, he used a Gaussian model to describe the distribution of errors in minimum temperature forecasts for Uppsala. He then determined the probability of frost (minimum temperature below 0°C) for various specific temperature forecasts. Ångström also described the impact that an increase in the accuracy of the forecasts would have on the frost probability estimates and then emphasized the point that a frost warning should be issued when this probability exceeded the ratio a/b .

In section 3, Ångström discussed the difficulties faced by a forecaster who must issue a categorical warning. Such a forecaster must "consider not if a certain occurrence of weather is probable or not, but more specially the relation between the probability of the event and the costs of damage and protection. Especially of the latter he (i.e., the forecaster) generally has a very indefinite idea." He goes on to point out that substantial differences of opinion exist about the (average) value of the ratio a/b even in particular situations and that the choice of an appropriate value "demands detailed knowledge of conditions . . . falling generally far on the side of the special knowledge of a forecaster of weather." Ångström then made the im-

portant additional point that "(w)hat makes the matter still more complicated is the fact that a/b has very different values in different cases."

As a result of these considerations, Ångström arrived at the following conclusion: "The most appropriate system seems therefore to be to leave to the clients concerned by the warning to form an idea of the value of a/b and to issue the warnings in such a form that the larger or smaller probability of the event gets clear from the formulation. The client may then himself consider if it is worth while to make arrangements of protection or to disregard a given warning."

He then indicated that the Swedish Weather Service was taking some first steps in this direction by issuing rain forecasts using a numerical scale. The numbers defining this scale correspond to particular probabilities of rain or showers. A similar, although less explicitly probabilistic, format was also being used for frost forecasts.

Finally, Ångström addressed another possible objection to the use of probabilities in forecasts, namely, the concern that since forecasters generally believe strongly in their (categorical) forecasts, they would seldom issue forecasts involving intermediate probability values. To counter this argument, he pointed out that (a priori) probabilities of rain (for example) assigned by forecasters should cluster around the probability value corresponding to the (a posteriori) fraction of correct categorical forecasts of rain. In those situations in which this fraction falls in the range of intermediate probability values, the forecasters presumably would and should make fairly frequent use of such intermediate probabilities.

In conclusion, Ångström expressed thanks to A. Wallén, Director of SMHA, for several consultations and to "C. G. Rossby, B.Sc., for a couple of good advices in reading the manuscript."

5. Discussion

In this section an attempt is made to put Ångström's papers in proper historical context and to discuss the significance of these papers in the context of probability forecasting and forecast use/value, as these topics are understood today. In addition, the relevance of these papers for related issues of current interest, in particular, ensemble forecasting and the provision of specialized weather services, is considered briefly.

Cooke (1906a,b) is usually credited with publishing the first papers on probability forecasting. His arguments in favor of quantifying the uncertainty inherent in weather forecasts were based on the belief that uncertainty in forecasts varied from day to day and that forecasters were capable of assessing this uncer-

tainty on an ordinal scale. Moreover, he believed that communication of this information to users could help to dispel the widespread notion that all forecasts were equally uncertain and thereby enhance the credibility of forecasts. Cooke also presented some encouraging empirical results related to probability forecasts made in Western Australia.

Cooke's seminal contributions were followed by papers by von Myrbach (1913) and Hallenbeck (1920). von Myrbach, evidently independently of Cooke, also advocated the use of numbers to characterize the degree of uncertainty in forecasts. Hallenbeck reported the results of an experiment in which probability forecasts were provided to farmers in the southwestern United States. Reference sometimes is made to papers by Besson (1905) and Rolf (1917) in this context; however, although these authors developed simple statistical forecasting methods that produced probabilistic forecasts, they were apparently not advocates of probability forecasting. In fact, their forecasts were usually converted immediately into categorical forecasts. No other significant papers related to probabilistic weather forecasting appeared until the 1940s.

Bilham (1922) generally is believed to have made the first significant contribution to the literature on the economic value of weather forecasts. He considered a simple protect/do not protect decision-making problem similar in structure to that studied by Ångström and developed an expression for the conditions under which it is advantageous to base decisions on yes/no forecasts (as opposed to always or never taking protective action). It should be noted that earlier papers by Peirce (1884) and Fujiwara and Nakamura (1919), although primarily concerned with the problem of forecast verification in 2×2 situations, also considered forecast value briefly or indirectly. Peirce introduced an expression for the "average profit per prediction" under restrictive assumptions concerning the outcomes (in 2×2 situations) involving gains or losses. Consideration of forecast value in the Fujiwara and Nakamura paper was limited to the arbitrary assignment of differential weights ("points") to the four possible outcomes in such situations. As far as the authors are aware, no other contributions related to this topic appeared in the published literature until the 1930s.

In our view, the significance of Ångström's papers, vis-a-vis the other early literature on these topics and subsequent developments in these areas, can be summarized briefly as follows:

1) Ångström's first paper contains a particularly insightful discussion of the major sources of uncertainty in weather forecasting. His description of the impact that small differences in initial conditions can have on the subsequent evolution of the atmosphere

is especially noteworthy. In particular, Ångström's example of a cone balancing on its top bears remarkable resemblance to other examples introduced 50–75 years later in connection with discussions of atmospheric predictability. For instance, Lorenz (1985) considered an example involving the behavior of balls placed within bowl-shaped depressions or placed at the top of dome-shaped hills in introducing an investigation of atmospheric stability and instability, and Palmer (1993) referred to the example of "a pencil balancing exactly on its tip" in connection with a discussion of the chaotic behavior of the atmosphere.

2) In describing a method of arriving at a probabilistic forecast, Ångström initially makes use of a relative frequency interpretation of probability. However, recognizing the practical limitations of this approach, he indicates (in the second paper) that this probability could also be assessed by a forecaster on the basis of his/her experience, intuition, judgment, etc. In these comments, he provides a heuristic argument for a subjective interpretation of probability, an interpretation in which probability is viewed as a subjective degree of belief (Winkler 1972; Murphy 1985). This argument (albeit brief) is particularly interesting because it predates the formal "redevelopment" of the subjective interpretation of probability in the late 1920s and early 1930s.

3) Ångström makes a particularly cogent argument regarding resolution of the problem of when and in what form storm warnings (or, by implication, other forecasts of this type) should be issued. If the warning is to be expressed in a categorical format, then the forecaster must know both the probability of the event and the critical value of the user's economic parameter (the ratio a/b in Ångström's papers) to decide whether or not to issue the warning. However, as Ångström points out, forecasters generally possess little if any information regarding the values of such economic parameters, and in any case, these values differ from user to user, location to location, etc. As a result, he concludes that the "most appropriate" solution is, in effect, to have users assess their own critical parameter values and to include information regarding the likelihood of occurrence of the relevant events in the forecasts. Thus, Ångström recommends that the task of arriving at the best possible forecast consistent with the forecaster's (or forecast system's) state of knowledge should be separated from the task of assessing the user's loss (or payoff) function—a clear, logical solution that is consistent with the theory of decision making under uncertainty (e.g., Raiffa 1968). Unfortunately, the widespread dissemination of forecasts expressed in an unqualified categorical format, including warnings of severe weather (see Murphy 1991), together with the meager amount of

reliable information generally available concerning users' loss functions, suggests that a substantial segment of the weather forecasting community has yet more than 70 years later to recognize the logic of this solution!

As noted previously, these two remarkable papers by Ångström also contain other noteworthy contributions. For example, he identified the ratio of the protection cost to the "risked value" as a parameter that could be used to describe the economic structure of idealized users' decision-making problems, and he produced quantitative expressions for the value (and "effectivity") of weather forecasts. Ångström's contributions also include the use of the Gaussian distribution both to predict the probability of frost and to model the errors in minimum temperature forecasts.

It should be noted as well that Ångström's expression for forecast value E in (5) may represent the first explicit example of a so-called quality/value relationship for weather forecasts. Given the economic parameters a and b , this expression specifies forecast value E as a function of forecast quality, as defined in this case by the conditional probabilities P_1 and P_3 . For recent discussions of this topic, see Katz and Murphy (1994) and Murphy and Ehrendorfer (1994).

For reasons of historical accuracy it is important to keep the early papers by Ångström and others in mind when assessing subsequent developments with regard to topics such as probability forecasting and forecast use/value. In the case of probability forecasting, the inspiration for and expectations for success in various experimental and operational programs conducted in the 1950–70 period in the United States find their roots in the early work of Cooke, Ångström, Hallenbeck, and others in the intervening years. Relatively comprehensive reviews of the status of probability forecasting as of the mid-1980s, as well as rather extensive lists of references, can be found in Murphy and Winkler (1984) and Murphy (1985). Probabilistic weather forecasts are now produced on an operational basis in many countries around the world, and these forecasts are more and more frequently provided directly to users (including the general public).

The last 15–20 years have also witnessed an increased interest in problems and issues related to forecast use/value. This interest has led to a variety of studies ranging from surveys of user groups regarding (inter alia) the use of forecasts to case studies of forecast use and forecast value in the context of prototypical and real-world decision-making problems. For an in-depth treatment of this topic, see Katz and Murphy (1994).

It is interesting to note that some particularly significant developments in these areas have taken place in

recent years in Ångström's native land of Sweden. Since at least the early 1980s many programs involving probability forecasting and the provision of specialized weather forecasts to users have been initiated at SMHI (e.g., Ivarsson et al. 1986). Moreover, in terms of their in-depth and continuing interactions with users, these programs have gone well beyond the usual forecaster–user or producer–consumer discussions (e.g., Kempe 1990; Liljas 1984; Wahlbin 1984; Wendt 1993).

In the 1980s it became increasingly clear that forecasts of various features of atmospheric flow patterns and some weather variables produced directly by numerical weather prediction models possessed positive "skill" beyond 3–5 days, but that this skill varied substantially from situation to situation. Attempts to differentiate between relatively high-skill and low-skill situations were undertaken initially under the rubric "forecasting forecast skill" (e.g., Hoffmann and Kalnay 1983) and involved analyzing the spread of multiple forecasts started from similar initial conditions. More recently, ensemble forecasting (i.e., forecasting in which an ensemble of forecasts is produced, each started from slightly different but appropriately chosen initial conditions) has focused instead on producing probability forecasts of various weather patterns and events defined in terms of specific values of weather variables at particular locations (e.g., Murphy 1990; Palmer et al. 1992; Tracton and Kalnay 1993). Although the formulation of meaningful and useful probabilistic forecasts in the context of ensemble forecasting undoubtedly involves significant new challenges, recognition of the need for and desirability of quantifying this uncertainty as it varies from situation to situation can be traced directly back to the work of Ångström, Cooke, and Hallenbeck in the first quarter of this century and, more recently, to Epstein (1969) and his colleagues, who developed the theoretical basis for the stochastic–dynamic approach to weather prediction.

In recent years, demands by end users and others (e.g., the media) for forecasts of a variety of variables on a broad spectrum of timescales have increased. In some countries (e.g., the United States), this demand has led to a substantial increase in the number of private firms providing forecasting services. In other countries (e.g., in many countries in Europe), it has led to the development of greatly enhanced commercial activities within public weather services. In these latter countries, this trend has been strongly influenced by substantial decreases in direct government support to the national weather services. For such services to be successful, whether undertaken in the public or private arena, the providers of the services must become intimately acquainted with the weather-sensitive ac-

tivities of the end users. In this regard, it is interesting to note that Ångström underlines the need to obtain quantitative information regarding the value of a/b (the economic parameter of concern). As he notes, it is not possible to determine the economic value (effectivity) of forecasts without such knowledge, nor is it possible to determine when warnings should and should not be issued in those circumstances in which the warnings must be expressed in a nonprobabilistic format. Unfortunately, the meteorological community as a whole has failed to heed Ångström's advice (and that proffered by others in the intervening years). Despite substantially increased contact with a broad spectrum of users, especially in recent decades, quantitative knowledge concerning the loss or payoff functions (e.g., cost–loss ratios) of end users remains very limited.

6. Conclusion

This paper has described and discussed briefly the contents of two remarkable papers by Anders Ångström on the topics of probability forecasting and the use/value of forecasts. These papers, which were rediscovered only recently, clearly represent important milestones in the early history and development of these two interrelated topics. They constitute the first published works (to the authors' knowledge) to provide explicit justification for the quantification of uncertainty in forecasts in terms of forecast use/value. In particular, they expose the underlying connection between the type of forecast required to characterize the forecaster's or forecast system's state of knowledge and the type of information needed by users to make efficient and effective decisions.

Especially noteworthy features of Ångström's papers include 1) a discussion of the sources of uncertainty in weather forecasting, 2) discourses on the problems of estimating probabilities by means of empirical relative frequencies and forecasters' subjective judgments, 3) a Gaussian model that is used to describe the accuracy of minimum temperature forecasts, 4) expressions for the economic value of weather warnings, 5) a quantitative analysis of the problems faced by forecasters in deciding whether or not to issue weather warnings when they are uncertain about future weather conditions, and 6) arguments concerning the need to obtain estimates of the costs and losses that may be incurred by potential users in order to assess the economic effectiveness of forecasts. In each instance, these discussions, expressions, etc., either appear to be the first treatments of the issues or build substantially and in a quantitative manner on earlier qualitative treatments.

We believe that it is important to keep in mind these early contributions by Ångström and others, as well as contributions in the intervening years, when evaluating recent and current developments in these areas. For example, it is clear that the roots of the argument that all forecasts should be accompanied by descriptions of their inherent uncertainty, which has been used recently to provide motivation for ensemble forecasting, can be traced back directly to the early work and writings of Cooke, Ångström, and Hallenbeck. While they used weather forecasts as examples, their arguments were clearly applicable to forecasts of all types including the output of models (recall that these papers were written long before operational numerical weather prediction became a reality).

Moreover, Ångström's comments to the effect that the operational meteorological community should acquire detailed information concerning users' costs and losses are noteworthy, especially in view of current efforts to provide more specialized weather services and to enhance the revenue from such services. As Ångström indicates, this information is needed in order to (inter alia) decide whether or not to issue forecasts in those situations in which the forecast products must be expressed in a nonprobabilistic format, as well as to provide essential input into the process of estimating the economic benefits of forecasts.

In conclusion, the rediscovery of Ångström's papers raises an interesting question. Do other early papers on these or related topics, perhaps in the form of unpublished manuscripts or in a language other than English, remain to be discovered? Dedicated individuals who persevere in their efforts to find such papers may be rewarded with additional insights into possible solutions to present-day problems.

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